Quantum Entanglement Transformations via Local Operations and Classical Communication

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CERTIFICATE OF ORIGINAL AUTHORSHIP

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Abstract

The primary goals of this thesis are two-fold: i) calculating the optimal entanglement transformation rate between two multipartite pure states via stochastic local operations and classical communication (SLOCC), and ii) showing the properties of a common resource for a set of multi-partite pure state via local operations and classical communication (LOCC) or SLOCC.

We introduce a notion of entanglement transformation rate to characterize the asymptotic comparability of two multi-partite pure entangled states under SLOCC. For two well known SLOCC inequivalent three-qubit states: Greenberger-Horne-Zeilinger (GHZ) state and W state, we show that the entanglement transformation rate from GHZ state to W state is exactly 1. We then apply similar techniques to obtain a lower bound on the entanglement transformation rates from an N-partite GHZ state to a class of Dicke states.

Then, we discuss the common resource for a set of pure states. We have completely solved the bipartite pure states case by explicitly constructing a unique optimal common resource state for any given set of states via LOCC. In the multi-partite setting, the general problem becomes quite complicated, and we focus on finding non-trivial common resources for the whole multi-partite state space of given dimensions. We show that $|GHZ_3\rangle = (1/\sqrt{3})(|000\rangle + |111\rangle + |222\rangle)$ is a nontrivial common resource for three-qubit systems via LOCC. We also obtain a number of interesting properties of non-trivial common resource states for two N-qubit pure states and multi-partite systems via SLOCC.

Key words: Entanglement Transformation, LOCC, GHZ state, W state, Tensor Rank, Common Resource States.

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